Part 1: No partial credit. Among the given choices for each problem, ONLY ONE is the correct answer. Either circle the correct choice or write it in the parenthesis. If you circle one choice but write down another in the parenthesis, the answer in the parenthesis will be considered your final answer. Each question is worth 8 points.

1. (   ) $L[y] = y'' - 3y' + 2y$. Evaluate $L[e^{-t}]$.

   (a) $-2e^{-t}$;

   (b) 0;

   (c) $4e^{-t}$;

   (d) $6e^{-t}$;

   (e) $8e^{-t}$.

2. (   ) Find the largest open interval for which the initial value problem

$$(t-2)y'' + \frac{y}{t-4} - \frac{t}{t-1} = 0, \quad y(3) = 0,\ y'(3) = 1,$$

   has a solution.

   (a) $0 < t < 4$;

   (b) $1 < t < 4$;

   (c) $2 < t < 4$;

   (d) $1 < t < \infty$;

   (e) $2 < t < \infty$.

3. (   ) Which one is NOT a solution for the differential equation

$$y'' + 3y' + 2y = 0.$$

   (a) e^{-t};

   (b) e^{t};

   (c) $-e^{-t}$;

   (d) $-e^{-2t}$;

   (e) $10e^{-t}$.

4. (   ) Find a proper form of the particular solution of the differential equation

$$y''' - 3y'' + 3y' - y = 3e^t.$$

(a) Ae^t;

(b) At^2e^t;

(c) At^3e^t;

(d) $(A\cos t + B\sin t)t^2e^t$;

(e) $(At + B)t^3e^t$.

5. (   ) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t, & t \ge 1 \end{cases}.$$

(a) $\dfrac{e^{-s}}{s^2}$;

(b) $\dfrac{e^{s}}{s^2}$;

(c) $e^s(\dfrac{1}{s^2} + \dfrac{1}{s})$;

(d) $e^{-s}(\dfrac{1}{s^2} + \dfrac{1}{s})$;

(e) $e^{-s}(\dfrac{1}{s^2} - \dfrac{1}{s})$.

6. (   ) The value of the constant r such that $y = x^r$ solves $x^2y'' + xy' - 2y = 0$ for $x > 0$ are:

(a) $\pm\sqrt{2}$;

(b) $\pm i\sqrt{2}$;

(c) $1 \pm \sqrt{2}$;

(d) $-1, -2$;

(e) $1, -2$.

7. ( ) Find the inverse Laplace transform of

$$\frac{e^{-s}}{s(s+1)}.$$

(a) $u_1(t)(1-e^{-t})$;

(b) $u_1(t)(1+e^{-(t-1)})$;

(c) $u_1(t)(e^{-(t-1)})$;

(d) $u_1(t)(1-e^{-(t-1)})$;

(e) $u_1(t)(1+e^{t-1})$.

8. ( ) If one solution of $y''+y'-2y=f(x)$ is $y(x)=\ln x$, then the general solution is:

(a) $c_1 \ln x$;

(b) $c_1 \ln x + c_2e^{x} + c_3e^{-2x}$;

(c) $c_1e^{-x} + c_2e^{2x} + \ln x$;

(d) $c_1e^{x} + c_2e^{-2x}$;

(e) $c_1e^{x} + c_2e^{-2x} + \ln x$.

9. ( ) Find the inverse Laplace transform of

$$\frac{s}{s^2-4s+8}.$$

(a) $\frac{1}{2}e^{2t}\sin(2t)$;

(b) $e^{2t}\cos(2t)$;

(c) $\frac{1}{2}e^{2t}\cos(2t)$;

(d) $e^{2t}(\cos(2t)+\sin(2t))$;

(e) $e^{2t}(\cos(2t)+2\sin(2t))$.

Part 2. Partial credit section. Show all your work neatly and concisely, and indicate your final answer clearly. If you simply write down the final answer without appropriate intermediate steps, you may not get full credit for that problem.

10. (12 points) Determine the general solution of the following equation:

$$y''' - y'' + y' - y = 0.$$

11. (16 points) Find the Laplace transform $Y(s) = \mathcal{L}\{y\}$ of the solution of the given initial value problem:

$$y'' + 4y = 4u_2(t), \qquad \text{with } y(0) = 0 \text{ and } y'(0) = 0.$$

(You don't need to find $y(t)$.)